Note on generalized $q$–Bernstein Durrmeyer operators

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Abstract: In this note, we discuss the results obtained in [6] for generalized $q$–Bernstein Durrmeyer operators. Here, we mention the readers about the correct form so that further research on this topic may be done correctly.

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1. Introduction

In 1987, the rapid development of $q$–calculus led to the discovery of new generalization of Bernstein polynomials by Lupas (see [5]), which involves $q$–integers. About after 10 years, in 1997 Phillips et al. [8] introduced another generalization of Bernstein polynomials based on $q$–integers and $q$–binomial coefficients. In 2005, Derriennic [3] proposed the first integral modification of q-Bernstein polynomials using jacobi weight. Gupta [4] in 2008 proposed the simple $q$–Bernstein-Durrmeyer operators and established some convergence properties in real domain. He estimated the moments of $q$–Bernstein-Durrmeyer operators and studied the rate of convergence for these operators. He established a direct results in terms of modulus of continuity. For detailed study on the applications of $q$– calculus in approximation theory and specially on the convergence of various $q$ operators, we refer the readers to the important recent book [1].

In 2014, Agrawal et al [2], introduced Stancu type generalization of modified Schurer operators based on $q$–calculus. They established the Voronovskaja type theorem for $q$–modified Schurer-Stancu operators. They also discussed local direct results for the operators and studied the statistical convergence of operators. In last section they obtained some convergence properties of the limit $q$–modified Schurer Stancu operators. Recently, Mishra and Patel [6] claimed to propose the Stancu type generalization of $q$–Bernstein Durrmeyer operators. Their operators is merely a special case of $q$–modified Schurer-Stancu operators of Agrawal et al. [2], which was introduced and studied earlier. This fact motivated us to write the present note.
2. Discussion

Agrawal et al. [2], introduced $q$-modified Schurer-Stancu operators as below:

**Definition 1.** For any $f \in C[0, 1 + p]$ and $\alpha, \beta, p \in \mathbb{N}_0$ (the set of all non-negative integers), $0 \leq \alpha \leq \beta$ and $x \in [0, 1 + p]$, the operator is defined by

$$S_{n,p}^{(\alpha, \beta)}(f, q, x) = \frac{[n+p+1]_q}{(1+p)^{2n+2p+1}} \sum_{k=0}^{n+p} b^{q}_{n+p,k}(x) q^{-k} \int_0^{1+p} f \left( \frac{[n]_q t + \alpha}{[n]_q + \beta} \right) b^{q}_{n+p,k}(qt) dq t,
$$

(2.1)

here $b^{q}_{n+p,k}(x) = \binom{n}{k}_q x^k (1 + p - x)^{n+p-k}$.

Later Mishra and Patel [6] claimed to introduce the following form of operators:

**Definition 2.** For any $f \in C[0, 1]$ and $0 \leq \alpha \leq \beta$, the operator is defined by

$$D_{n,q}^{\alpha,\beta}(f; x) = [n+1]_q \sum_{k=0}^{n} p_{nk}(q; x) q^{-k} \int_0^1 f \left( \frac{[n]_q t + \alpha}{[n]_q + \beta} \right) p_{nk}(q; qt) dq t, \tag{2.2}
$$

here $p_{nk}(q; x) = \binom{n}{k}_q x^k (1 - x)^{n-k}$.

Clearly operator (2.2) can be obtained from operator (2.1) by replacing $p$ with 0.

We want to bring notice on the limiting case of the operator $D_{n,q}^{\alpha,\beta}(f; x)$. Limiting case of any operator must be independent of the variable $n$. Limiting case proposed in [6] involves $n$ which is not correct. We can redefine the limiting operator in better way as below:

For $q \in (0, 1)$ be fixed and $x \in [0, 1]$,

$$D_{\infty,q}^{\alpha,\beta}(f; x) = \frac{1}{1-q} \sum_{k=0}^{\infty} p_{\infty k}(q; x) q^{-k} \int_0^1 f \left( \frac{t + \alpha(1-q)}{1+\beta(1-q)} \right) p_{\infty k}(q; qt) dq t, \tag{2.3}
$$

where $p_{\infty k}(q; x) = \frac{1}{(1-q)^k [n]_q} x^k (1-x)^{\infty}$. Again operator (2.3) can be obtained as special case (for $p = 0$) of operator $S_{\infty,p}^{(\alpha, \beta)}(f, q, x)$ [2, Definition 3].

In approximation theory estimation of moments of the operator is initial and most important objective, all the analysis of the operator depends on moments. By using the methods as discussed in [4, pp. 175], the moments...
of the operator (2.3) takes the following form:

\[
D_{\infty,q}^{\alpha,\beta}(f; x) = 1,
\]

\[
D_{\infty,q}^{\alpha,\beta}(t; x) = \frac{1}{1 + \beta(1 - q)}(1 + q(x - 1)) + \frac{\alpha(1 - q)}{1 + \beta(1 - q)},
\]

\[
D_{\infty,q}^{\alpha,\beta}(t; x) = \frac{1}{(1 + \beta(1 - q))^2}\left((1 - q)^2(1 + q) + q(1 + 2q)(1 - q)x + q^3(1 - q)x + q^4x^2\right)
\]
\[
+ \frac{2\alpha(1 - q)}{1 + \beta(1 - q)^2}(1 + q(x - 1)) + \frac{\alpha^2(1 - q)^2}{(1 + \beta(1 - q))^2}.
\]

3. Conclusion

The generalization proposed by Mishra and Patel [6] is just a special case of generalization of Durrmeyer operators introduced by Agrawal et al [2] for \(p = 0\). Analysis of both papers are same, their is no new result discussed by Mishra and Patel. Moreover, limiting case of the operator proposed in [6] is not appropriate. To see the correct form of the more general case we refer [2]. Also the similar mistake was done by Mishra and Patel for limiting operators in [7].

References